# **Syllabus for Abstract Mathematics**

A course for the undergraduate students on specialization Mathematics and Economics

*Lecturer: Alexei Akhmetshin*

*Class teacher: V Shmarov*

**Course description**

Abstract Mathematics is a two-semester course for the third year students studying at ICEF specializing in Mathematics and Economics. It is based on the Introduction to Abstract Mathematics course of the University of London (UoL) with further expansions into selected topics from algebra, real analysis and topology.

The emphasis of the course is on the theory rather than on the method. One central topic of the course is formal mathematical reasoning. We will practise in formulating precise mathematical statements and proving them rigorously. These skills are essential for the current specialization, although they often remain in shadows in other math courses where the focus is on solving problems through calculation.

The second central topic of the course is the abstract mathematical structures from algebra (groups, fields, etc.), analysis and topology (topological spaces, manifolds). We will develop some of these theories roughly to the extent of standard 1st and 2nd-year courses of the mathematical departments.

Upon completion of this course the students will have to take the University of London (UoL) exam at the end of the fourth semester of their studies at ICEF.

**Learning objectives**

Having taken this course you should

* have a knowledge of main mathematical concepts in discrete mathematics, algebra, real analysis and topology;
* be able to use formal notations correctly and in connection with precise statements in English;
* be able to formulate statements of the key theorems and present their proofs;
* be able to find and formulate proofs of problems based on those theorems.

**Teaching Methods**

The course program consists of:

* lectures,
* classes,
* regular self-study based on class problem sets, regular homework assignment problem sets and extra problem sets.

**Assessment and grade determination**

There are the following forms of control:

* written home assignments posted and turned in every week;
* written exams at the end of modules 1, 2 and 3.
* University of London exam by the end of module 4 on Abstract Mathematics MA103.

The cumulative final grade is comprised of:

* average grade for the home assignments (20%);
* Exams at the end of modules 1, 2 and 3 (40%);
* UoL external exam (40%).

**Reading**

Recommended by UoL Abstract Mathematics syllabus:

1. Biggs, Norman L. Discrete mathematic, 2nd edition. (Oxford University Press, 2002).
2. Eccles, P.J., *An* *Introduction to Mathematical Reasoning: numbers, sets and functions*. (Cambridge University Press, 1997).
3. Binmore, K.G., *Mathematical Analysis: A Straightforward Approach.* (Cambridge University Press, 1982).
4. Bryant, V., *Yet Another Introduction to Analysis* (Cambridge University Press, 1982).
5. Halmosh, P.R., *Finite Dimentional Vector Spaces*. (Springer, 1996).
6. Rudin, W., *Principles of Mathematical Analysis*, 3rd edition, (McGraw-Hill, 1976).

Additional reading:

1. Vinberg E.B., *A Course in* *Algebra.* (Factorial Press, 2001).
2. Warner, Frank W., *Foundations of Differentiable Manifolds and Lie Groups*. (Springer, 1983).
3. Vassiliev V.A., *Introduction to Topology*. (MCCME 2014).

**Course outline**

1. Foundations
   1. Mathematical statements, proof, logic and sets.
   2. Natural numbers and proof by induction.
   3. Functions and counting.
   4. Equivalence relations and the integers.
   5. Divisibility and prime numbers.
   6. Congruence and modular arithmetic.
   7. Rational, real and complex numbers.
2. Algebra
   1. Algebraic structures: groups, rings, fields.
   2. Group theory: subgroups, quotient groups and homomorphisms, cosets & Lagrange’s theorem.
   3. Group actions.
   4. Direct and Semidirect products and Abelian Groups.
   5. Modules and Vector spaces.
   6. Field Theory and Galois theory\*.
   7. Introduction to the representation theory of finite groups.\*
3. Analysis and Topology
   1. Topological spaces and operations with them.
   2. Homotopy groups, homotopy equivalence.
   3. Coverings.\*
   4. Cell spaces (CW-complexes).\*
   5. Fiber bundles\*.
   6. Differentiable manifolds.
   7. Tangent vectors and differentials.
   8. Submanifolds, Diffeomorphisms.
   9. Inverse Function Theorem, Implicit Function Theorem.
   10. Vector Fields.\*
   11. Differential Forms\*.
4. Lie Groups\*

Note: topics marked with \* will be covered if time permits.