Marketing via Friends: Strategic Diffusion of Information in Social Networks with Homophily

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Homophily

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\[
\rho = 0
\]
\[
\rho = 0.5
\]
\[
\rho = 0.75
\]
\[
\rho = 1.0
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Literature review

- **WOM literature:**
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- **Homophily:**
  - Friendship and segregation: Currarini, Jackson & Pin (2009)
  - Learning and diffusion: Jackson & Golub (2010)
  - Social norms and preferences: Christakis & Fowler (2007), Fiore and Donath (2005)
Model

**Network structure:**

There is measure $\gamma$ of consumers of type $A$ and $1 - \gamma$ of type $B$. Consumers are embedded into an undirected network of social contacts:

- **Degree distribution $p(k)$**
- **Vector $(\rho_A, \rho_B)$** identifies proportion of consumers of the same type in the neighborhood of a randomly chosen consumer of type $A$ and $B$. 

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Model cont’d

Consumers:

- Heterogenous preferences towards the product
  - Across types:
    - Type A prefers high values of characteristic w
    - Type B prefers low values of characteristic w
  - Within types:
    - Reservation price $\bar{P}_j$
    - Threshold level of the product’s characteristic $\bar{w}_j$, s.t. induces a consumer to buy the product

Consumers can buy the product only if they learn about it from:

- Direct advertisement.
- Observing a neighbor who has already acquired the product.
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Monopolist:

Knows degree distribution $p(k)$ and homophily levels $(\rho_A, \rho_B)$.

Maximizes profits by choosing:

- Price $P \in [0, 1]$
- Characteristic of the product $w \in [0, 1]$

Cost of production is 0.

To induce sales the monopolist advertises product to infinitesimal part of the population.
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- $\bar{P}_j$ and threshold value of characteristic $\bar{w}_j$ are i.i.d. $U[0,1]$
- Consumers buy the product with probability:
  - Type $A$: $q_A = \Pr(w \geq \bar{w}_j \cap P \leq \bar{P}_j) = (1 - P)(1 - w)$
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Illustration of the monopolist problem

- Monopolist chooses optimally characteristic \( w \) and price \( P \) to maximize profits:

\[
w = 0. \quad 0 \leq P \leq 1
\]

Preferences frontier

Induced network of potential buyers
Monopolist chooses optimally characteristic $w$ and price $P$ to maximize profits:

$$w = 0.25$$

Preferences frontier  Induced network of potential buyers
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- Monopolist chooses optimally characteristic $w$ and price $P$ to maximize profits:

\[ w = 0.5 \]

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\[
q^A = \frac{1 - P}{P} \\
q^B = \frac{0.75}{1 - P}
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![Preferences frontier](image1)

![Induced network of potential buyers](image2)
Cascade of sales per advertisement

- We modify Newman’s mixing patterns model by incorporating consumers decision to buy the product (probability that a node is operational).
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- Expected size of cascade of sales per advertisement:

\[
s(q^A, q^B, \rho, z_1, z_2, \gamma) =
\]

\[
(\gamma 1 - \gamma) \left[ I + \frac{z_2}{z_1} \left( I - \frac{z_2}{z_1} \begin{pmatrix} q^A \rho & q^A(1 - \rho) \\ q^B(1 - \rho) & q^B \rho \end{pmatrix} \right)^{-1} - I \right] \begin{pmatrix} q^A \\ q^B \end{pmatrix}
\]

where \(z_1\) and \(z_2\) are expected numbers of first and second neighbors.
The global cascade phase

- **The global cascade:** advertisement to infinitesimal part of the population leads to acquisition of the product by non-zero proportion of the population.
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Proposition

If $\frac{z_2}{z_1} > \min\{2, \rho^{-1}\}$ there exist combinations of product characteristic and price $(w, P)$ such that global cascade of sales arises.
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If \( \frac{z_2}{z_1} > \min\{2, \rho^{-1}\} \) there exist combinations of product characteristic and price \((w, P)\) such that global cascade of sales arises.

- Necessary condition for existence of the giant component of connected consumers, \( \frac{z_2}{z_1} > 1 \).
- The existence of the global cascade in the case when \( \frac{z_2}{z_1} < 2 \) hinges on the homophily level.
Optimal design strategy:

Proposition

- The optimal characteristic of the product is the following correspondence:

\[
w^* = \begin{cases} 
[0, 1], & \rho = \frac{1}{2} \\
1/2, & \rho < \frac{1}{2} \\
\{0, 1\}, & \rho > \frac{1}{2}
\end{cases}
\]
Optimal design strategy: Intuition

$\rho = 0$
- All neighbors are of different type.
- Spreading depends on the attractiveness of the product to both types.

$\rho = 1$
- Two clusters of consumers of the same type.
- Specialized design is optimal.
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Optimal pricing strategy:

Proposition

- The optimal price $P^*$ is lower than in the case of full information and for $\rho > \frac{1}{2}$ is strictly decreasing function in the level of homophily.

- For two degree distributions $p(k)$ and $p'(k)$ and corresponding optimal prices $P^*$ and $P'^*$ if $p(k)$ is a mean preserving spread of $p'(k)$ then $P^* < P'^*$. 
Demand function

\[ Q(P, \rho, z_1, z_2) = \begin{cases} 
\frac{1-P}{2} \left( 1 + \frac{z_1(1-P)}{2-z_2/z_1(1-P)} \right), & \rho \leq \frac{1}{2} \\
\frac{1-P}{2} \left( 1 + \frac{z_1(1-P)}{\frac{1}{\rho}-z_2/z_1(1-P)} \right), & \rho > \frac{1}{2}
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Proposition

The demand function \( Q(P, \rho, z_1, z_2) \) has following properties:

1. Decreasing and convex in price \( P \).
2. Increasing and convex in homophily level \( \rho \), for \( \rho > \frac{1}{2} \).
3. The absolute value of the price elasticity of demand is:

\[ \frac{P}{1-P} \left(1 + z_1 \left(\frac{1}{z_1 - (1-P)z_2\rho} - \frac{1}{z_1 + (1-P)(z_1^2 - z_2)\rho}\right)\right), \]

which is higher than price elasticity in the case of full information \( \frac{P}{1-P} \) and is increasing in homophily level \( \rho \), for \( \rho > \frac{1}{2} \).
Demand: Intuition

- Demand is decreasing and convex in $P$.

Induced network of buyers.
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- Monopolist surplus is increasing in the level of homophily
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\[ PS(P^*(\rho), \rho, z_1, z_2) = P^*(\rho) \times Q(P^*(\rho), \rho, z_1, z_2) \]
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*Consumers and producers surplus are increasing functions in the homophily level of the society.*

- **Monopolist surplus is increasing in the level of homophily**
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PS(P^*(\rho), \rho, z_1, z_2) = P^*(\rho) \times Q(P^*(\rho), \rho, z_1, z_2)
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- **Consumer surplus is increasing in the level of homophily**
  - Demand is increasing - more consumers buy the product.
  - The optimal price is decreasing in the level of homophily.

\[
CS(P^*(\rho), \rho, z_1, z_2) = \int_{P^*(\rho)}^{1} Q(P, \rho, z_1, z_2) dP
\]
Model Extensions

- **Targeted advertisement.**
  - Targeting advertisement is always optimal.
  - For high enough connectivity the optimal design is the same as without targeting.
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  - Bend inward frontier - results are the same.
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- **Monopolist benefits from one group.**
  - Low levels of homophily - compromise design is still optimal.
  - High levels of homophily - compromise design is optimal when audience is small.
Selling to one type.

- Price $P$ is fixed and the monopolist maximizes sales to consumers of type $B$. 

Proposition

There is threshold value $\hat{\rho}(z_1, z_2)$ such that if $\rho < \hat{\rho}(z_1, z_2)$ the optimal characteristic $w^* = 0$, while if $\rho > \hat{\rho}(z_1, z_2)$ then $w^* = 1$. 

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**Proposition**

There is threshold value $\hat{\rho}_1(z_1, z_2)$ such that if $\rho < \hat{\rho}_1(z_1, z_2)$ the optimal characteristic $w_1^*$ belongs to the interval $\left(0, \frac{1}{2}\right]$, while if $\rho > \hat{\rho}_1(z_1, z_2)$ then $w_1^* = 0$.
Selling to one type.

- Consumers of type $A$ constitute 80% of the population ($\gamma = 0.8$). The monopolist maximizes sales to consumers of type $B$.

- The solution:

```
<table>
<thead>
<tr>
<th>$\rho^A$</th>
<th>$w^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.80</td>
</tr>
<tr>
<td>0.80</td>
<td>0.85</td>
</tr>
<tr>
<td>0.85</td>
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Conclusions

For low levels of homophily the compromise design of product is preferred to specialized products even if there is no cost of producing more than one type of product.

Price elasticity of demand is increasing in the homophily level.

Monopolist and consumers benefit from increase in the level of homophily.

A product designed to attract both types of consumers may be optimal even if a monopolist benefits only from one group of consumers.
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